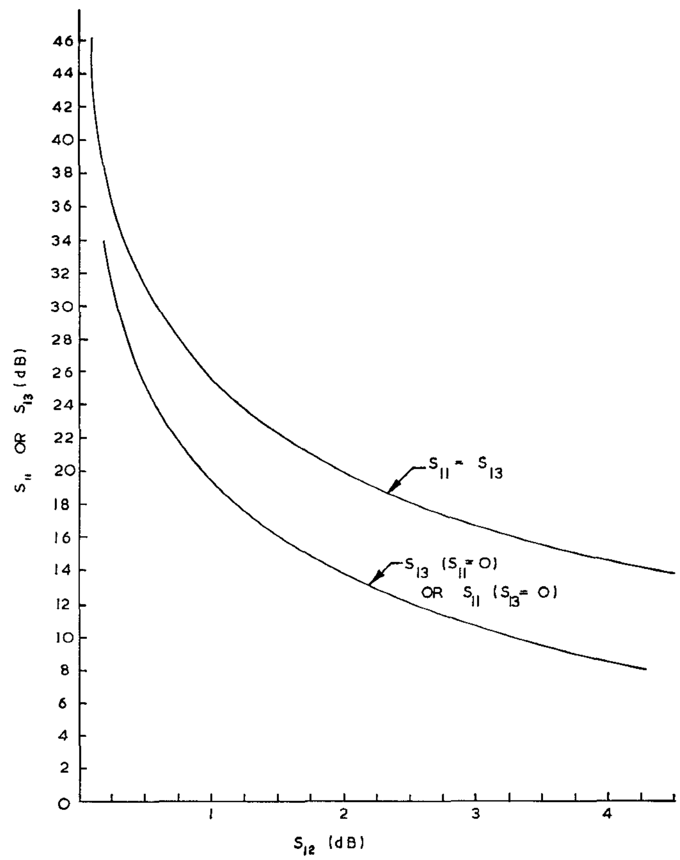

Fig. 4. Relation between S parameters of semi-ideal circulators.

where it has been assumed that one has symmetrical splitting:

$$\theta_{-1} = -\theta_{+1}. \tag{61}$$

In semi-ideal circulators, S_{11} and S_{13} are completely determined by S_{12}, provided it is assumed that the amplitudes of s_{+1} and s_{-1} are equal and the splitting is symmetrical. This means that the latter quantity can be obtained simply by measuring either S_{11} or S_{13}.

The first case to be considered is the one in which the angle between s_{+1} and s_{-1} is such that $S_{11}=0$. This condition is obtained by setting $S_{11}=0$ in (58). The relation between S_{12} and S_{13} is given graphically in Fig. 4.

The second case to be considered here is the one in which the angle between s_{+1} and s_{-1} is such that $S_{13}=0$. This condition is obtained by setting $S_{13}=0$ in (60). The relation between S_{11} and S_{12} is shown in Fig. 4.

VII. Conclusions

The relation between the dissipation and scattering eigenvalues in lossy junctions has been given. The results have been used to directly construct the scattering matrices of a number of 3-port lossy junctions. These results can also be applied to junctions with unequal dissipation eigenvalues that lead to asymmetric frequency responses of the scattering parameters.

Acknowledgment

The author would like to thank Dr. Y. Konishi for some helpful comments.

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Correction to "Scattering by a Ferrimagnetic Circular Cylinder in a Rectangular Waveguide"

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In the above paper,[1] $\sin^{-1} t$ in (17) (Section III-B, p. 524) should be interpreted as $\pi - \mathrm{Sin}^{-1}\, t$, where $\mathrm{Sin}^{-1}\, t$ denotes the principal value of $\sin^{-1} t$. Therefore, (23) and (24) should read as follows:

$$S_{11} = \sum_{n=-\infty}^{\infty} (-1)^n A_n \frac{4}{\beta_1 a} \sin\left(\frac{\pi x_0}{a} - n\,\mathrm{Sin}^{-1}\frac{\pi}{k_0 a}\right) \tag{1}$$

and

$$S_{21} = 1 + \sum_{n=-\infty}^{\infty} A_n \frac{4}{\beta_1 a} \sin\left(\frac{\pi x_0}{a} + n\,\mathrm{Sin}^{-1}\frac{\pi}{k_0 a}\right) \tag{2}$$

respectively. Accordingly, Table I should be modified as shown.

The corrected numerical evaluation of $|S_{11}|^2 + |S_{21}|^2$ shows that the unitary condition of the S-matrix is satisfied within a roundoff error for any value of parameters. This is due to the fact that the unitary conditions of the S-matrix is guaranteed for any size of truncation in our formulation of the paper [1]. The following is a proof of this property. The electric field outside the post is expressed as follows:

$$E_z = E_0 \sum_{n=-\infty}^{\infty} J_n(\rho_0^+) e^{-jn\theta_0^+} \sin\left(\frac{\pi x_0}{a} + n\alpha\right) + E_0 \sum_{n=-\infty}^{\infty} A_n \sum_{s=-\infty}^{\infty} \left[H_n^{(2)}(\rho_s^+) e^{-jn\theta_s^+} - (-1)^n H_n^{(2)}(\rho_s^-) e^{jn\theta_s^-}\right]. \tag{3}$$

Consider a region enclosed by two contours $ABCDA$ and F, as shown in Fig. 1. Application of the two-dimensional Poynting theorem to this region yields

$$\mathrm{Re}\left[\frac{1}{2}\int_{AB} (-E_z H_x^*) dx + \frac{1}{2}\int_{DC} E_z H_x^* dx\right] + \mathrm{Re}\left[\frac{1}{2}\oint_F E_z H_\theta^* dl\right] = 0. \tag{4}$$

On the contour AB and DC far from the post, E_z in (3) can be rewritten in the form

$$E_z = E_0 \sin\frac{\pi x}{a} \cdot (e^{-j\beta y} + S_{11} e^{j\beta y}) \tag{5}$$

and

$$E_z = E_0 \sin\frac{\pi x}{a} \cdot S_{21} e^{-j\beta y} \tag{6}$$

respectively. Substitution of (5) and (6) and their corresponding

Manuscript received June 12, 1972.
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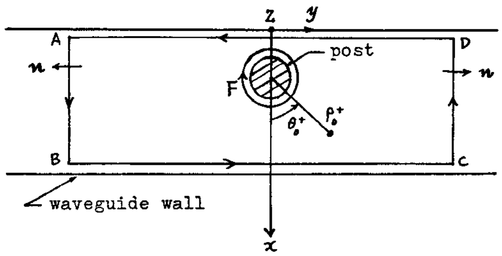

Fig. 1. Contours along which Poynting integrals are evaluated.

TABLE I

COMPUTED VALUES FOR $|S_{11}|^2+|S_{21}|^2$

Ω_H	k_0R = 0.05	k_0R = 0.1
0.57	0.9999999	0.9999999
0.58	0.9999999	0.9999999
0.59	0.9999999	0.9999999
0.60	0.9999999	0.9999999

TABLE II

COMPUTED VALUES FOR S_{11} AND S_{21} WHEN $k_0R=0.05$

((1) and (4) stand for $\sum_{n=-1}^{1}$ and $\sum_{n=-4}^{4}$, respectively)

Ω_H	S_{11}	S_{21}	
0.584	0.32193 ∠-73.892°	0.94676 ∠16.107°	(1)
	0.32193 ∠-73.892°	0.94676 ∠16.107°	(4)
0.586	0.99955 ∠-0.96237°	0.029818 ∠89.037°	(1)
	0.99955 ∠-0.96237°	0.029818 ∠89.037°	(4)
0.588	0.27685 ∠71.257°	0.96091 ∠-18.742°	(1)
	0.27685 ∠71.257°	0.96091 ∠-18.742°	(4)

magnetic field components into the first term on the left-hand side of (4) yields

$$\frac{E_0^2\beta a}{4\omega\mu_0}\{|S_{11}|^2+|S_{21}|^2-1\} = -\mathrm{Re}\left[\frac{1}{2}\oint_F E_zH_\theta^*dl\right]. \quad (7)$$

On the contour F, E_z can be rewritten in the form

$$E_z = E_0\sum_p \left[J_p(v)\sin\left(\frac{\pi x_0}{a}+p\alpha\right)+J_p(v)\sum_n A_nh_{np} + A_pH_p^{(2)}(v)\right]\cdot e^{-jp\theta} \quad (8)$$

and then the corresponding magnetic field component is given by

$$H_\theta = \frac{E_0}{j\omega\mu_0}\sum_p\left[vJ_p'(v)\sin\left(\frac{\pi x_0}{a}+p\alpha\right)+vJ_p'(v)\sum_n A_nh_{np} + A_pvH_p^{(2)\prime}(v)\right]\cdot\frac{e^{-jp\theta}}{R} \quad (9)$$

where $v=k_0R$. Boundary conditions lead to equations

$$J_p(v)\sin\left(\frac{\pi x_0}{a}+p\alpha\right)+J_p(v)\sum_n A_ph_{np}+A_pH_p^{(2)}(v) = B_pJ_p(u)$$

$$vJ_p'(v)\sin\left(\frac{\pi x_0}{a}+p\alpha\right)+vJ_p'(v)\sum_n A_nh_{np}+A_pvH_p^{(2)\prime}(v) = B_p[MuJ_p'(u)-KpJ_p(u)]. \quad (10)$$

Hence from (7), (8), (9), and (10), it is obtained that

$$\frac{\beta a}{4\omega\mu_0}\{|S_{11}|^2+|S_{21}|^2-1\} = -\mathrm{Re}\left\{\frac{\pi}{j\omega\mu_0}\sum_n |B_n|^2J_n(u)\cdot[MuJ_n'(u)-KnJ_n(u)]\right\} = 0. \quad (11)$$

This proves that the unitary condition always holds for any size of (10).

As can be seen from the preceding discussion, the unitary condition cannot be used as a check for error due to the truncation. In our formulation of the paper [1], only the numerical convergence of S_{11} and S_{21} through (31) serve as a check.

To see numerical convergence of S_{11} and S_{21}, a comparison of $\sum_{n=-1}^{1}$ and $\sum_{n=-4}^{4}$ has been made around the dipolar resonant point $\Omega=\Omega_H+\frac{1}{2}$, which appears in case of vanishingly small diameter of a ferrimagnetic cylinder (see Table II). Here, k_0R is set to 0.05 as in [1].

From these results it is seen that a solution for $\sum_{n=-1}^{1}$ has sufficient accuracy as far as the dipole resonance is concerned.